Mathematics

45) Find the roots of the quadratic equation $2x^2 - 3x - 1 = 0$ using quadratic formula.

Soln:-

$2x^2 - 3x - 1 = 0$

$a = 2, \quad b = -3, \quad c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times -1}}{2 \times 2}$

$x = \dfrac{3 \pm \sqrt{9 + 8}}{4}$

$x = \dfrac{3 \pm \sqrt{17}}{4}$

$x = \dfrac{3 + \sqrt{17}}{4} \quad \text{or} \quad x = \dfrac{3 - \sqrt{17}}{4}$

46)

Prove that : $\dfrac{\cos\theta - \sin\theta \cdot \cos\theta}{\cos\theta + \sin\theta \cdot \cos\theta} = \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1}$.

Ans:-

$\text{LHS} = \dfrac{\cos\theta - \sin\theta \cdot \cos\theta}{\cos\theta + \sin\theta \cdot \cos\theta}$

$= \dfrac{\cos\theta(1 - \sin\theta)}{\cos\theta(1 + \sin\theta)}$

$= \dfrac{1 - \sin\theta}{1 + \sin\theta}$

$= \dfrac{1 - \dfrac{1}{\operatorname{cosec}\theta}}{1 + \dfrac{1}{\operatorname{cosec}\theta}}$

$= \dfrac{\dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta}}{\dfrac{\operatorname{cosec}\theta + 1}{\operatorname{cosec}\theta}}$

$= \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + 1} = \text{RHS}$

47) A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	Number of students
0-6	11
6-10	10
10-14	7
14-20	4
20-28	4

Soln:

Find the midpoint of the given interval using the formula.

Midpoint (x_i) = (upper limit + lower limit)/2

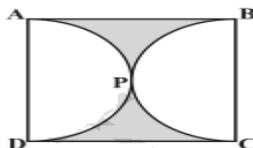| Class interval	Frequency (f_i)	Mid-point (x_i)	$f_i x_i$
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68
20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
	Sum f_i = 40		**Sum $f_i x_i$ = 499**

The mean formula is,

Mean = $\bar{x} = \sum f_i x_i / \sum f_i$

$= 499/40 = 12.48$ days

Therefore, the mean number of days a student was absent = 12.48.

48) Find the area of the shaded region in Fig. if ABCD is a square of side 14 cm and APD and BPC are semicircles.

Soln: Side of the square ABCD (as given) = 14 cm

So, Area of ABCD = a^2

$= 14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2$

We know that the side of the square = diameter of the circle = 14 cm

So,

side of the square = diameter of the semicircle = 14 cm

∴ Radius of the semicircle = 7 cm

Now, area of the semicircle = $(\pi R^2)/2$

$= (22/7 \times 7 \times 7)/2 \text{ cm}^2$

$= 77 \text{ cm}^2$

∴ Area of two semicircles = $2 \times 77 \text{ cm}^2 = 154 \text{ cm}^2$

Hence, area of the shaded region = Area of the Square – Area of two semicircles = $196 \text{ cm}^2 - 154 \text{ cm}^2$

$= 42 \text{ cm}^2$

49) Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

Ans:-

$p(x, y) = \left(\dfrac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \dfrac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$

$A(4, -3), \qquad B(8, 5), \quad m_1 : m_2 = 3 : 1$

$(x_1, y_1) \qquad\qquad (x_2, y_2)$

$x = \dfrac{3(8) + 1(4)}{3 + 1}, \qquad y = \dfrac{3(5) + 1(-3)}{3 + 1}$

$x = \dfrac{24 + 4}{4}, \qquad y = \dfrac{15 - 3}{4}$

$x = \dfrac{28}{4}, \quad y = \dfrac{12}{4}$

$x = 7, \qquad y = 3$

The co-ordinates of the required point P (x, y) is (7, 3)