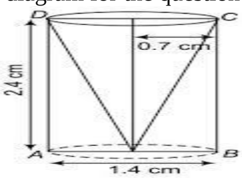
Mathematics

Surface Areas and Volumes

EXERCISE 12.1

8) From a solid cylinder whose height is 2.4 cm and diameter is 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Ans: The diagram for the question is as follows:



From the question, we know the following:
The diameter of the cylinder = diameter of conical cavity = 1.4 cm
So, the radius of the cylinder = radius of the conical cavity = 1.4/2 = 0.7   Also, the height of the cylinder = height of the conical cavity = 2.4 cm

$\therefore$ Slant height of the conical cavity $(l) = \sqrt{h^2 + r^2}$

$$= \sqrt{(2.4)^2 + (0.7)^2}$$

$$= \sqrt{5.76 + 0.49} = \sqrt{6.25}$$

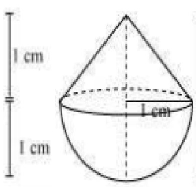
$$= 2.5 \text{ cm}$$

Now, the TSA of the remaining solid = surface area of conical cavity + TSA of the cylinder
$= \pi rl + (2\pi rh + \pi r^2)$
$= \pi r(l + 2h + r)$
$= (22/7) \times 0.7(2.5+4.8+0.7)$
$= 2.2 \times 8 = 17.6 \text{ cm}^2$
So, the total surface area of the remaining solid is 17.6 cm^2

Exercise: 12.2

1. A solid is in the shape of a cone standing on a hemisphere, with both their radii being equal to 1 cm and the height of the cone being equal to its radius. Find the volume of the solid in terms of π.

Soln:
Here r = 1 cm and h = 1 cm.
The diagram is as follows.



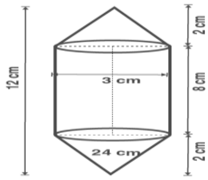
Now, Volume of solid = Volume of conical part + Volume of hemispherical part
We know the volume of cone = $\frac{1}{3}\pi r^2 h$
And,
The volume of the hemisphere = $\frac{2}{3}\pi r^3$
So, the volume of the solid will be

$$= \frac{1}{3}\pi(1)^2[1 + 2(1)] \text{cm}^3 = \frac{1}{3}\pi \times 1 \times [3] \text{cm}^3$$

$= \pi \text{ cm}^3$

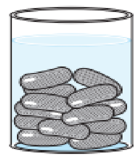
2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3cm, and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model are nearly the same.)

Soln:

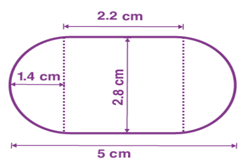


Given, Height of cylinder = 12 – 4 = 8 cm
Radius = 1.5 cm
Height of cone = 2 cm
Now, the total volume of the air contained will be =
= Volume of cylinder+2×(Volume of the cone)
$\therefore$ Total volume $= \pi r^2 h + [2 \times (\frac{1}{3}\pi r^2 h)]$
$= 18\pi + 2(1.5\pi)$
$= 66 \text{ cm}^3$.

3. A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and a diameter of 2.8 cm (see figure).



Solution:



It is known that the gulab jamuns are similar to a cylinder with two hemispherical ends.
So, the total height of a gulab jamun = 5 cm.
Diameter = 2.8 cm
So, radius = 1.4 cm
$\therefore$ The height of the cylindrical part =
= 5 cm–(1.4+1.4) cm
= 2.2 cm
Now, the total volume of one gulab jamun =
=Volume of cylinder + Volume of two hemispheres
$= \pi r^2 h + (4/3)\pi r^3$
$= 4.312\pi + (10.976/3)\pi$
$= 25.05 \text{ cm}^3$
We know that the volume of sugar syrup = 30% of the total volume
So, the volume of sugar syrup in 45 gulab jamuns =
$= 45 \times 30\% (25.05 \text{ cm}^3)$
$= 45 \times 7.515 = 338.184 \text{ cm}^3$