MATHEMATICS : *Circles*

Exercise: 10.2

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm, respectively (see Fig. 10.14). Find the sides AB and AC.

Ans: **(….Contd)**

$48x(14+x) = (56+4x)^2$

$48x = [4(14+x)]^2/(14+x)$

$48x = 16(14+x)$

$48x = 224+16x$

$32x = 224$

$x = 7$ cm

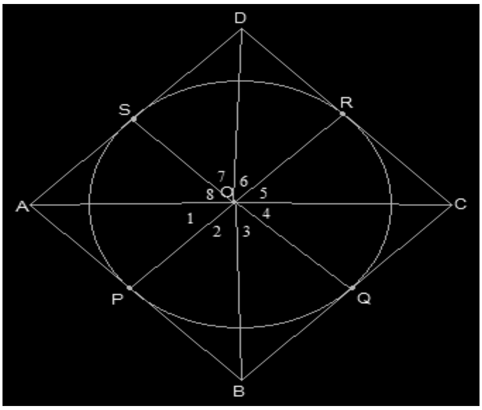
So, AB = 8+x

i.e. AB = 15 cm

And, CA = x+6 =13 cm

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: First, draw a quadrilateral ABCD which will circumscribe a circle with its centre O in a way that it touches the circle at points P, Q, R, and S. Now, after joining the vertices of ABCD, we get the following figure:



Now, consider the triangles OAP and OAS.

AP=AS (They are the tangents from the same point A)

OA = OA (It is the common side)

OP = OS (They are the radii of the circle)

So, by SSS congruency △OAP ≅ △OAS

So, ∠POA = ∠AOS

Which implies that∠1 = ∠8

Similarly, other angles will be

∠4 = ∠5

∠2 = ∠3

∠6 = ∠7

Now by adding these angles, we get

∠1+∠2+∠3 +∠4 +∠5+∠6+∠7+∠8 = 360°

Now by rearranging,

(∠1+∠8)+(∠2+∠3)+(∠4+∠5)+(∠6+∠7) = 360°

2∠1+2∠2+2∠5+2∠6 = 360°

Taking 2 as common and solving, we get

(∠1+∠2)+(∠5+∠6) = 180°

Thus, ∠AOB+∠COD = 180°

Similarly,

it can be proved that ∠BOC+∠DOA = 180°

Therefore, the opposite sides of any quadrilateral which is circumscribing a given circle will subtend supplementary angles at the centre of the circle.

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AREAS RELATED TO CIRCLES

Exercise: 11.1

1. Find the area of a sector of a circle with a radius 6 cm if the angle of the sector is 60°.

Soln: It is given that the angle of the sector is 60°

We know that the area of sector = $(\theta/360°)\times\pi r^2$

∴ area of the sector with angle 60° =

$= (60°/360°)\times\pi r^2$ cm^2

$= (36/6)\pi$ cm^2

$= 6\times22/7$ cm^2 = 132/7 cm^2

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Soln: Circumference of the circle, C= 22 cm (given)

It should be noted that a quadrant of a circle is a sector which is making an angle of 90°.

Let the radius of the circle = r

As C = 2πr = 22,

R = 22/2π cm = 7/2 cm

∴ area of the quadrant = $(\theta/360°) \times \pi r^2$

Here, θ = 90°

So, A = $(90°/360°) \times \pi\, r^2$ cm^2

= (49/16) π cm^2

= 77/8 cm^2 = 9.6 cm^2

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Soln: Length of minute hand = radius of the clock (circle)

∴ Radius (r) of the circle = 14 cm (given)

Angle swept by minute hand in 60 minutes = 360°

So, the angle swept by the minute hand in 5 minutes = 360° × 5/60 = 30°

We know,

Area of a sector = $(\theta/360°) \times \pi r^2$

Now, the area of the sector making an angle of 30° = $(30°/360°) \times \pi r^2$ cm^2

$= (1/12) \times \pi 14^2$

= (49/3)×(22/7) cm^2

= 154/3 cm^2

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) minor segment

(ii) major sector. (Use π = 3.14)

Soln:

(Contd….)