ಎಸ್‌ಎಸ್‌ಎಲ್‌ಸಿ–ಇಂಗ್ಲಿಷ್ ಮಾಧ್ಯಮ

MATHEMATICS : *Circles*

Exercise: 10.1

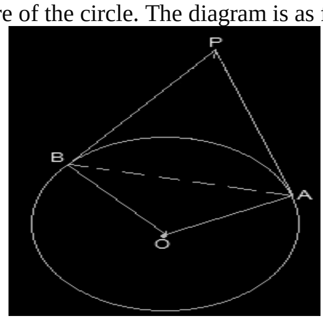
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: **(…….Contd)**

Taking 2 as common and solving, we get
$(\angle 1+\angle 2)+(\angle 5+\angle 6) = 180°$
Thus, $\angle AOB+\angle COD = 180°$
Similarly, it can be proved that $\angle BOC+\angle DOA = 180°$
Therefore, the opposite sides of any quadrilateral which is circumscribing a given circle will subtend supplementary angles at the centre of the circle.

14. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

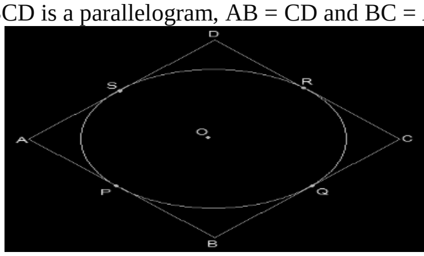
Ans: First, draw a circle with centre O. Choose an external point P and draw two tangents, PA and PB, at point A and point B, respectively. Now, join A and B to make AB in a way that subtends $\angle AOB$ at the centre of the circle. The diagram is as follows:



From the above diagram, it is seen that the line segments OA and PA are perpendicular.
So, $\angle OAP = 90°$
In a similar way, the line segments $OB \perp PB$ and so, $\angle OBP = 90°$
Now, in the quadrilateral OAPB,
$\therefore \angle APB+\angle OAP+\angle PBO+\angle BOA = 360°$ (since the sum of all interior angles will be 360°)
By putting the values, we get,
$\angle APB + 180° + \angle BOA = 360°$
So, $\angle APB + \angle BOA = 180°$ (Hence proved).

15. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans: Consider a parallelogram ABCD which is circumscribing a circle with a centre O. Now, since ABCD is a parallelogram, AB = CD and BC = AD.

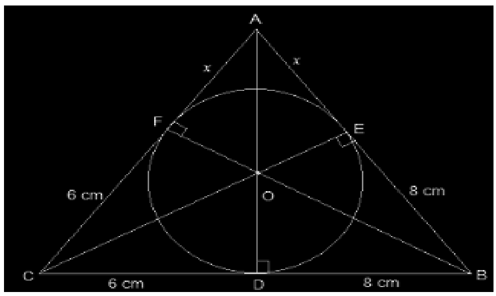


From the above figure, it is seen that,
(i) DR = DS
(ii) BP = BQ
(iii) CR = CQ
(iv) AP = AS

These are the tangents to the circle at D, B, C, and A, respectively.
Adding all these, we get
DR+BP+CR+AP = DS+BQ+CQ+AS
By rearranging them, we get
(BP+AP)+(DR+CR) = (CQ+BQ)+(DS+AS)
Again by rearranging them, we get
AB+CD = BC+AD
Now, since AB = CD and BC = AD, the above equation becomes
2AB = 2BC
$\therefore$ AB = BC
Since AB = BC = CD = DA, it can be said that ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm, respectively (see Fig. 10.14). Find the sides AB and AC.

Ans: The figure given is as follows:



Consider the triangle ABC,
We know that the length of any two tangents which are drawn from the same point to the circle is equal.
So,
(i) CF = CD = 6 cm
(ii) BE = BD = 8 cm
(iii) AE = AF = x
Now, it can be observed that,
(i) AB = EB+AE = 8+x
(ii) CA = CF+FA = 6+x
(iii) BC = DC+BD = 6+8 = 14
Now the semi-perimeter "s" will be calculated as follows
2s = AB+CA+BC
By putting the respective values, we get,
2s = 28+2x
s = 14+x

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

By solving this, we get,
$= \sqrt{(14+x)48x}$ ……… (i)
Again, the area of $\triangle ABC = 2 \times$ area of ($\triangle AOF + \triangle COD + \triangle DOB$)
$= 2\times[(\frac{1}{2}\times OF\times AF)+(\frac{1}{2}\times CD\times OD)+(\frac{1}{2}\times DB\times OD)]$
$= 2\times\frac{1}{2}(4x+24+32) = 56+4x$ …………..(ii)
Now from (i) and (ii), we get,
$\sqrt{(14+x)48x} = 56+4x$
Now, square both sides, *(Contd…….)*