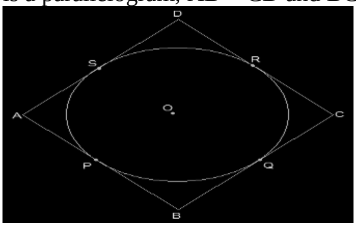
ಎಸ್‌ಎಸ್‌ಎಲ್‌ಸಿ–ಇಂಗ್ಲಿಷ್ ಮಾಧ್ಯಮ

MATHEMATICS : *Circles*

Exercise: 10.1

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans: Consider a parallelogram ABCD which is circumscribing a circle with a centre O. Now, since ABCD is a parallelogram, AB = CD and BC = AD.



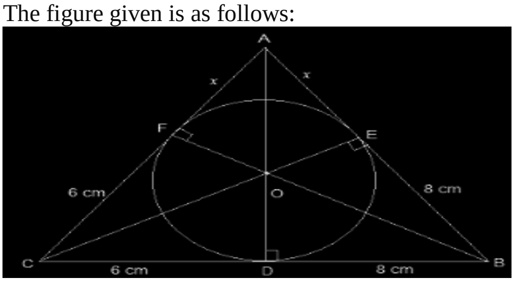
From the above figure, it is seen that,

(i) DR = DS
(ii) BP = BQ
(iii) CR = CQ
(iv) AP = AS

These are the tangents to the circle at D, B, C, and A, respectively.
Adding all these, we get
DR+BP+CR+AP = DS+BQ+CQ+AS
By rearranging them, we get
(BP+AP)+(DR+CR) = (CQ+BQ)+(DS+AS)
Again by rearranging them, we get
AB+CD = BC+AD
Now, since AB = CD and BC = AD, the above equation becomes
2AB = 2BC
∴ AB = BC
Since AB = BC = CD = DA, it can be said that ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm, respectively (see Fig. 10.14). Find the sides AB and AC.

Ans:
The figure given is as follows:



Consider the triangle ABC,
We know that the length of any two tangents which are drawn from the same point to the circle is equal.
So,
(i) CF = CD = 6 cm
(ii) BE = BD = 8 cm
(iii) AE = AF = x
Now, it can be observed that,
(i) AB = EB+AE = 8+x
(ii) CA = CF+FA = 6+x
(iii) BC = DC+BD = 6+8 = 14
Now the semi-perimeter "s" will be calculated as follows
2s = AB+CA+BC
By putting the respective values, we get,
2s = 28+2x
s = 14+x

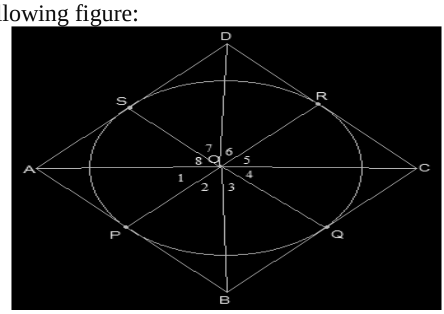
Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

By solving this, we get,
= √(14+x)48x ……… (i)
Again, the area of △ABC = 2 × area of (△AOF + △COD + △DOB)
= 2×[(½×OF×AF)+(½×CD×OD)+(½×DB×OD)]
= 2×½(4x+24+32) = 56+4x …………..(ii)
Now from (i) and (ii), we get,
√(14+x)48x = 56+4x
Now, square both sides,

$48x(14+x) = (56+4x)^2$
$48x = [4(14+x)]^2/(14+x)$
$48x = 16(14+x)$
$48x = 224+16x$
$32x = 224$
$x = 7$ cm
So, AB = 8+x
i.e. AB = 15 cm
And, CA = x+6 =13 cm.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: First, draw a quadrilateral ABCD which will circumscribe a circle with its centre O in a way that it touches the circle at points P, Q, R, and S. Now, after joining the vertices of ABCD, we get the following figure:



Now, consider the triangles OAP and OAS.
AP = AS(They are the tangents from the same point A)
OA = OA (It is the common side)
OP = OS (They are the radii of the circle)
So, by SSS congruency △OAP ≅ △OAS
So, ∠POA = ∠AOS
Which implies that∠1 = ∠8
Similarly, other angles will be
∠4 = ∠5
∠2 = ∠3
∠6 = ∠7
Now by adding these angles, we get
∠1+∠2+∠3 +∠4 +∠5+∠6+∠7+∠8 = 360°
Now by rearranging,
(∠1+∠8)+(∠2+∠3)+(∠4+∠5)+(∠6+∠7) = 360°
2∠1+2∠2+2∠5+2∠6 = 360°

(Contd…….)