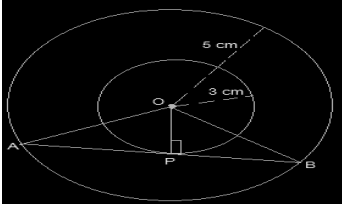


MATHEMATICS : Circles

Exercise: 10.1

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans: Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle, which touches the smaller circle at a point P, as shown in the figure below.



(....Contd)

$$AP = 4$$

Now, as $OP \perp AB$,

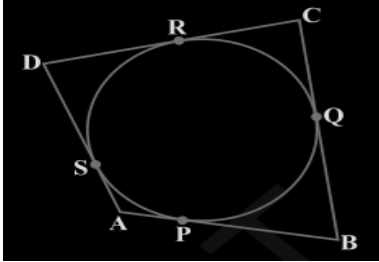
Since the perpendicular from the centre of the circle bisects the chord, AP will be equal to PB.

$$\text{So, } AB = 2AP = 2 \times 4 = 8 \text{ cm}$$

So, the length of the chord of the larger circle is 8 cm.

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that $AB + CD = AD + BC$

Ans: The figure given is:



From the figure, we can conclude a few points, which are

$$(i) DR = DS$$

$$(ii) BP = BQ$$

$$(iii) AP = AS$$

$$(iv) CR = CQ$$

Since they are tangents to the circle from points D, B, A, and C, respectively.

Now, adding the LHS and RHS of the above equations, we get,

$$DR + BP + AP + CR = DS + BQ + AS + CQ$$

By rearranging them, we get,

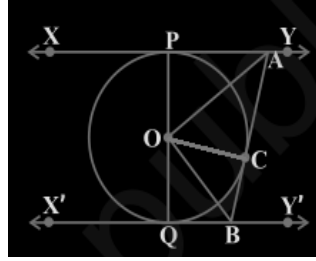
$$(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$$

By simplifying,

$$AD + BC = CD + AB$$

9. In Fig. 10.13, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with the point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.

Ans: From the figure given in the textbook, join OC. Now, the diagram will be as



Now, the triangles $\triangle OPA$ and $\triangle OCA$ are similar using SSS congruency as

(i) $OP = OC$ They are the radii of the same circle

(ii) $AO = AO$ It is the common side

(iii) $AP = AC$ These are the tangents from point A

So, $\triangle OPA \cong \triangle OCA$

Similarly,

$$\triangle OQB \cong \triangle OCB$$

So,

$$\angle POA = \angle COA \dots (\text{Equation} \dots \dots \dots i)$$

$$\text{And, } \angle QOB = \angle COB \dots (\text{Equation} \dots \dots \dots ii)$$

Since the line POQ is a straight line, it can be considered as the diameter of the circle.

$$\text{So, } \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$$

Now, from equations (i) and equation (ii), we get,

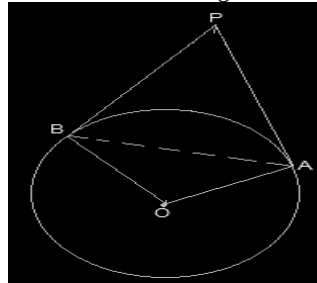
$$2\angle COA + 2\angle COB = 180^\circ$$

$$\angle COA + \angle COB = 90^\circ$$

$$\therefore \angle AOB = 90^\circ$$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Ans: First, draw a circle with centre O. Choose an external point P and draw two tangents, PA and PB, at point A and point B, respectively. Now, join A and B to make AB in a way that subtends $\angle AOB$ at the centre of the circle. The diagram is as follows:



From the above diagram, it is seen that the line segments OA and PA are perpendicular.

$$\text{So, } \angle OAP = 90^\circ$$

In a similar way, the line segments $OB \perp PB$ and so, $\angle OBP = 90^\circ$

Now, in the quadrilateral OAPB,

$$\therefore \angle APB + \angle OAP + \angle PBO + \angle BOA = 360^\circ \text{ (since the sum of all interior angles will be } 360^\circ)$$

By putting the values, we get,

$$\angle APB + 180^\circ + \angle BOA = 360^\circ$$

$$\text{So, } \angle APB + \angle BOA = 180^\circ \text{ (Hence proved).}$$

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans:

(Contd.....)