

MATHEMATICS : *Circles*

Exercise: 10.1

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° , then $\angle POA$ is equal to

- (A) 50° (B) 60°
(C) 70° (D) 80°

Ans: (.....Contd)

Now, consider the triangles $\triangle OPB$ and $\triangle OPA$.

Here,

$AP = BP$ (Since the tangents from a point are always equal)

$OA = OB$ (Which are the radii of the circle)

$OP = OP$ (It is the common side)

Now, we can say that triangles OPB and OPA are similar using SSS congruency.

$$\therefore \triangle OPB \cong \triangle OPA$$

$$\text{So, } \angle POB = \angle POA$$

$$\angle AOB = \angle POA + \angle POB$$

$$2(\angle POA) = \angle AOB$$

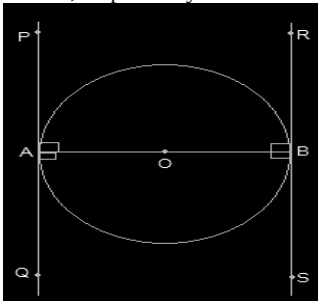
By putting the respective values, we get

$$\Rightarrow \angle POA = 100^\circ / 2 = 50^\circ$$

As the angle $\angle POA$ is 50° , option A is the correct option.

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans: First, draw a circle and connect two points, A and B, such that AB becomes the diameter of the circle. Now, draw two tangents, PQ and RS, at points A and B, respectively.



Now, both radii, i.e. AO and OB, are perpendicular to the tangents.

So, OB is perpendicular to RS, and OA is perpendicular to PQ.

$$\text{So, } \angle OAP = \angle OAQ = \angle OBR = \angle OBS = 90^\circ$$

From the above figure, angles OBR and OAQ are alternate interior angles.

Also, $\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$ (Since they are also alternate interior angles)

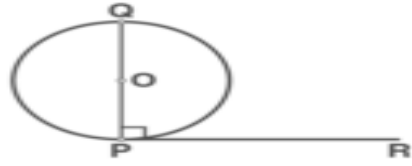
So, it can be said that line PQ and line RS will be parallel to each other (Hence Proved).

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans: Let, O is the centre of the given circle.

A tangent PR has been drawn touching the circle at point P.

Draw $QP \perp RP$ at point P, such that point Q lies on the circle.



$$\angle OPR = 90^\circ \text{ (radius } \perp \text{ tangent)}$$

$$\text{Also, } \angle QPR = 90^\circ \text{ (Given)}$$

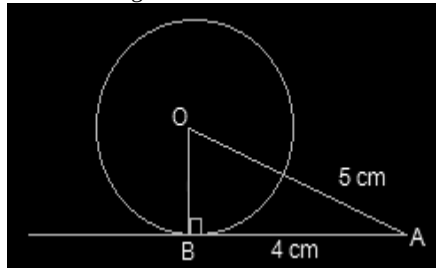
$$\therefore \angle OPR = \angle QPR$$

Now, the above case is possible only when centre O lies on the line QP.

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

6. The length of a tangent from point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Ans: Draw the diagram as shown below.



Here, AB is the tangent that is drawn on the circle from point A.

So, the radius OB will be perpendicular to AB, i.e., $OB \perp AB$

We know, $OA = 5\text{ cm}$ and $AB = 4\text{ cm}$

Now, In $\triangle ABO$,

$$OA^2 = AB^2 + BO^2 \text{ (Using Pythagoras' theorem)}$$

$$5^2 = 4^2 + BO^2$$

$$BO^2 = 25 - 16$$

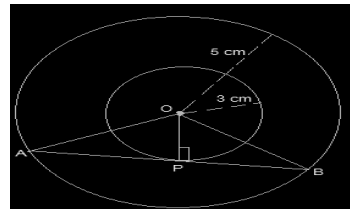
$$BO^2 = 9$$

$$BO = 3$$

So, the radius of the given circle, i.e., BO, is 3 cm.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans: Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle, which touches the smaller circle at a point P, as shown in the figure below.



From the above diagram, AB is tangent to the smaller circle at point P.

$$\therefore OP \perp AB$$

Using Pythagoras' theorem in triangle OPA,

$$OA^2 = AP^2 + OP^2$$

$$5^2 = AP^2 + 3^2$$

$$AP^2 = 25 - 9$$

(Contd..)