

MATHEMATICS Chapter-5

Arithmetic Progression

EXERCISE 5.3

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solⁿ:- (.....Contd)

Now, Sum of first n terms

$$S_n$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[2 \times 1 + (n-1)2]$$

$$= n[1 + n - 1]$$

$$= n(n)$$

$$= n^2$$

10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below :

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Sol :- (i) $a_n = 3 + 4n$

We have $a_n = 3 + 4n$

put $n=1, 2, 3, 4, \dots$ in Succession, we get

$$a_1 = 3 + 4(1) = 3 + 4 = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$a_3 = 3 + 4(3) = 3 + 12 = 15$$

$$a_4 = 3 + 4(4) = 3 + 16 = 19$$

∴

$$a_2 - a_1 = 11 - 7 = 4$$

$$a_3 - a_2 = 15 - 11 = 4$$

$$a_4 - a_3 = 19 - 15 = 4$$

i.e. $a_{k+1} - a_k$ is the same every time

So, $a_1, a_2, \dots, a_n, \dots$ from an AP

Here $a = a_1 = 7$

$$d = a_2 - a_1 = 4$$

Sum of the first 15 term = S_{15}

$$S_{15} = 49$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{15}{2}[2 \times a + (15-1)d]$$

$$\frac{15}{2}[2a + 14d] = 49$$

$$= 15(a + 7d)$$

$$= 15(7 + 7 \times 4)$$

$$= 15(7 + 28)$$

$$= (15)(35)$$

$$= 525$$

(ii) $a_n = 3 + 4n$

We have $a_n = 9 - 5n$

put $n=1, 2, 3, 4, \dots$ in Succession, we get

$$a_1 = 9 - 5(1) = 9 - 5 = 4$$

$$a_2 = 9 - 5(2) = 9 - 10 = -1$$

$$a_3 = 9 - 5(3) = 9 - 15 = -6$$

$$a_4 = 9 - 5(4) = 9 - 20 = -11$$

$$\therefore a_2 - a_1 = -1 - 4 = -5$$

$$a_3 - a_2 = -6 - (-1) = -6 + 1 = -5$$

$$a_4 - a_3 = -11 - (-6) = -11 + 6 = -5$$

i.e. $a_{k+1} - a_k$ is the same every time

So, $a_1, a_2, \dots, a_n, \dots$ from an AP

Here $a = a_1 = 4$

$$d = a_2 - a_1 = -5$$

Sum of the first 15 term = S_{15}

$$S_{15} = 49$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{15}{2}[2 \times a + (15-1)d]$$

$$\frac{15}{2}[2a + 14d] = 49$$

$$= 15(a + 7d)$$

$$= 15(4 + 7 \times (-5))$$

$$= 15(4 - 35)$$

$$= (15)(-31)$$

$$= -465$$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.

Soln:- We have

Sum of the first n term = $4n - n^2$

$$S_n = 4n - n^2$$

Put $n = 1$

$$S_1 = 4(1) - (1)^2$$

$$= 4 - 1 = 3$$

$$a_1 = 3$$

Hence the first term is 3

Put $n = 2$

$$S_2 = 4(2) - (2)^2$$

$$= 8 - 4 = 4$$

Hence the Sum of terms is 4

Second term

$$= S_2 - S_1$$

$$= 4 - 3 = 1$$

Put $n = 3$

$$S_3 = 4(3) - (3)^2$$

$$= 12 - 9 = 3$$

3rd term = $a_3 = S_3 - S_2$

$$= 3 - 4 = -1$$

Put $n = 9, 10$

$$S_9 = 4(9) - (9)^2 = 36 - 81 = -45$$

$$S_{10} = 4(10) - (10)^2 = 40 - 100 = -60$$

∴ 10th term = $a_{10} = S_{10} - S_9$

$$= -60 - (-45)$$

$$= -60 + 45 = -15$$

$$S_{n-1} = 4(n-1) - (n-1)^2$$

$$4n - 4 - (n^2 - 2n + 1)$$

$$4n - 4 - n^2 + 2n - 1 = 6n - n^2 - 5$$

n th term = a_n

$$S_n - S_{n-1}$$

$$(4n - n^2) - (6n - n^2 - 5)$$

$$= 5 - 2n$$