MATHEMATICS Chapter–1

Arithmetic Progression

EXERCISE 5.2

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Ans:- (……Contd)

Solving (1) and (2), we get

$a = -13$ d=5

So 1st term = -13

2nd term= - 13 +5 = - 8

3rd term= - 8 +5 = - 3

Hence the first three terms of the given AP are

-13. -8, and -3

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Soln:- Here, a = Rs 5000 d= Rs 200

l = Rs 7000

Suppose that his income reached Rs 7000 after n year

Then $l = a+(n-1)d$

$7000= 5000+ (n-1)200$

$(n-1)200 = 7000 - 5000$

$(n-1)200 = 2000$

$(n-1)= 2000/200=10$

$n= 10+1 = 11$

Hence, his income reached Rs 7000 in 11th Year

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the *n*th week, her weekly savings become Rs 20.75, find *n*.

Soln:Here, a=Rs 5 d= Rs1.75 a_n = Rs 20.75

We know that

$a_n = a + (n-1)d$

$20.75 = 5+ (n-1)(1.75)$

$(n-1)(1.75) = 20.75 - 5$

$(n-1)(1.75) = 15.75$

$(n-1)= 15.75/1.75$

$n-1= 9$

$n=9+1 = 10$

Hence, the required value of n is 10

Sum of First *n* Terms of an AP

1) Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, $a = 8$, $d = 3 - 8 = -5$, $n = 22$.

We know that

$$S = \frac{n}{2}[2a+(n-1)d]$$

$$Therfore \quad S = \frac{22}{2}[16+21(-5)]$$

$$=11(16-105)$$

$$=11(-89) = -979$$

So, the sum of the first 22 terms of the AP is – 979.

2) If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Solution : Here, S_{14} = 1050, n = 14, a = 10.

$$S = \frac{n}{2}[2a+(n-1)d]$$

$$1050 = \frac{14}{2}[20+13d]$$

$$=140+91d$$

$$910 = 91d$$

$$d = 10$$

$$Therefore \quad a_{20} = 10+(20-1)x10 = 200$$

i.e 20*th term* 200

3) How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Solution : Here, $a = 24$, $d = 21 - 24 = -3$, $S_n = 78$. We need to find n.

$$S = \frac{n}{2}[2a+(n-1)d]$$

$$78 = \frac{n}{2}[48+(n-1)(-3)]$$

$$= \frac{n}{2}[51-3n]$$

or $3n^2 - 51n + 156 = 0$

or $n^2 - 17n + 52 = 0$

or $(n - 4)(n - 13) = 0$

or $n = 4$ or 13

Both values of n are admissible. So, the number of terms is either 4 or 13.

4) Find the sum of

(i) the first 1000 positive integers

(ii) the first *n* positive integers

Solution :

(i) Let S = 1 + 2 + 3 + . . . + 1000

Using the formula $S_n = \frac{n}{2}[a+l]$ for the sum of the first n terms of an AP, we have

$$S_{1000} = \frac{1000}{2}(l+1000)$$

$$= 500x1001 = 500500$$

So, the sum of the first 1000 positive integers is 500500.

(ii) Let $Sn = 1 + 2 + 3 + . . . + n$

Here a = 1 and the last term l is n.

$$S_n = \frac{n(1+n)}{2} \quad or \quad S_n = \frac{n(n+1)}{2}$$

So, the sum of first n positive integers is given by

$$S_n = \frac{n(n+1)}{2}$$

5) Find the sum of first 24 terms of the list of numbers whose *n*th term is given by $a_n = 3 + 2n$

Solution : As $a_n = 3 + 2n$,

so, $a_1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 7$

$a_3 = 3 + 2 \times 3 = 9$

List of numbers becomes 5, 7, 9, 11, . . .

Here, 7 – 5 = 9 – 7 = 11 – 9 = 2 and so on.

So, it forms an AP with common difference $d = 2$.

Therefore, $S_{24} = \frac{24}{2}$ (2 x5+ (24 -1)x 2)

=12(10+46)= 627

So, sum of first 24 terms of the list of numbers is 672.

6) A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

(i) the production in the 1st year

(ii) the production in the 10th year

(iii) the total production in first 7 years

Soln : (i) Since the production increases uniformly by a fixed number every year,

the number of TV sets manufactured in 1st, 2nd, 3rd, . . ., years will form an AP.

Let us denote the number of TV sets manufactured in the nth year by an.

Then, $a_3 = 600$ and $a_7 = 700$

or, $a + 2d = 600$

and $a + 6d = 700$

Solving these equations, we get ***(Contd…)***