

MATHEMATICS Chapter-5

Arithmetic Progression

8. The angles of the triangle are in A. P. The smallest angle is 30° . Show that the triangle is a right angled triangle.

Soln:- (.....Contd)

\therefore the angles are $30^\circ, 60^\circ, 90^\circ$; hence the triangle is a right angled triangle as one of its angle is 90° .

9. The angles of a quadrilateral are in A. P. If the smallest angle is 15° , find the angles of the quadrilateral.

Soln:- Let  ABCD be a quadrilateral

Therefore let the angles be $a, a + d, a + 2d, a + 3d$ respectively.

We also know that the sum of the four angles of a quadrilateral is 360° .

Hence, $15^\circ + 15^\circ + d + 15^\circ + 2d + 15^\circ + 3d = 360^\circ$

$$60^\circ + 6d = 360^\circ$$

$$6d = 360^\circ - 60^\circ = 300^\circ$$

$$d = \frac{300}{6} = 50$$

\therefore the other angles of the quadrilateral are

$$15^\circ, 15^\circ + 50, 15^\circ + 50(2), 15^\circ + 50(3)$$

$$\Rightarrow 15^\circ, 15^\circ + 50, 15^\circ + 100, 15^\circ + 150$$

$$\Rightarrow 15^\circ, 65^\circ, 115^\circ, 165^\circ$$

10. Find the three numbers of an A. P whose sum is 12 and their product is 48.

Soln:- Given that the three numbers are in A. P.

Let the numbers be $(a - d), a, (a + d)$.

$$(a - d) + a + (a + d) = 12 \quad \dots\dots(i)$$

$$3a = 12$$

$$a = \frac{12}{3} = 4 \quad \dots\dots(ii)$$

$$(a - d) a (a + d) = 48 \quad \dots\dots(iii)$$

Substituting the value of a in (iii), we get

$$(4 - d) 4 (4 + d) = 48$$

$$(4 - d) (4 + d) = \frac{48}{4} = 12$$

$$4^2 - d^2 = 12 \Rightarrow 16 - d^2 = 12$$

$$\therefore d^2 = 16 - 12 = 4$$

$$\Rightarrow d = \pm 2$$

\therefore three numbers are 2, 4 and 6 (or) 6, 4 and 2.

To find nth term of an A.P

The nth term of the A.P, $a_n = a + (n - 1)d$

(1) Find a_n and a_{15} for the A.P 1, 4, 7, 10,

Soln:- The given A.P is 1, 4, 7, 10,

The first term ' a ' is 1, the common difference ' d ' is 3.

The nth term of an A. P is given by

$$a_n = a + (n - 1)d \quad \dots\dots(i)$$

Substituting the values of a and d in (i), we get

$$a_n = 1 + (n - 1)3$$

$$= 1 + 3n - 3$$

$$\therefore a_n = 3n - 2 \quad \dots\dots(ii)$$

Hence, by substituting for ' n ' as 15, we get -

$$a_{15} = 3(15) - 2$$

$$= 45 - 2 = 43$$

(2) Find a_{20} for the A.P 8, 2, -4, -10,

Soln:- The given A.P is 8, 2, -4, -10, ...

The first term ' a ' is 8, the common difference ' d ' is -6, ' n ' the number of terms is 20.

Substituting the values of a, n and d in

$$a_n = a + (n - 1)d; \text{ we get}$$

$$a_{20} = 8 + (20 - 1) - 6$$

$$= 8 + 19(-6) = -106$$

(3) Which term of the A.P : 21, 18, 15, is -81? Also, check whether any term is 0?

Ans:- The A.P : 21, 18, 15, is -81

The first term ' a ' is 21, the common difference ' d ' is $18 - 21 = -3$ and ' a_n ' the nth term is -81.

But the general eqn for finding the nth term of A.P is

$$a_n = a + (n - 1)d \quad \dots\dots(i)$$

Substituting the values of a, a_n and d in the above eqn we get,

$$-81 = 21 + (n - 1)(-3)$$

$$-81 = 21 - 3n + 3$$

$$-81 = -3n + 24$$

$$\Rightarrow 3n = 81 + 24$$

$$n = \frac{105}{3} = 35$$

Therefore, the 35th term of the given AP is 81.

To check whether any term of the A.P is 0, i.e., $a_n = 0$

Substituting the values of a, a_n and d in the eqn (i) we get,

$$0 = 21 + (n - 1)(-3)$$

$$0 = 21 - 3n + 3$$

$$0 = -3n + 24$$

$$\Rightarrow 3n = 24$$

$$n = \frac{24}{3} = 8$$

This shows that the 8th term is 0.

(4) The fifth and tenth are in the ratio 1 : 2 and $a_{12} = 36$, find d

Soln :- Given that $a_5 : a_{10} = 1 : 2$ and $a_{12} = 36$

$$\frac{a_5}{a_{10}} = \frac{1}{2}$$

$$\Rightarrow 2a_5 = a_{10}$$

$$\text{as } a_{12} = a_5 + 7d \text{ and } a_{12} = a_{10} + 2d$$

$$\Rightarrow 2(a_{12} - 7d) = (a_{12} - 2d)$$

$$2(36 - 7d) = (36 - 2d)$$

$$72 - 14d = 36 - 2d$$

$$\Rightarrow 12d = 36$$

$$\therefore d = \frac{36}{12} = 3$$

(5) Find the 10th term of the AP: 2, 7, 12,

Soln:- Here, $a = 2, d = 7 - 2 = 5$ and $n = 10$.

We have $a_n = a + (n - 1)d$

$$\text{So, } a_{10} = 2 + (10 - 1) \times 5 = 2 + 45 = 47$$

Therefore, the 10th term of the given AP is 47.