ಎಸ್ಎಸ್ಎಲ್ಸಿ–ಇಂಗ್ಲಿಷ್ ಮಾಧ್ಯಮ

MATHEMATICS Chapter–5

Arithmetic Progression

6) In which of the following situations, does the list of numbers involved make an arithmetic progression, and why? (i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1 /4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at8 % per annum ***(Exercise 5.1 problem)***

Soln:- (i) Taxi fare for 1km=Rs 15 = a_1

Taxi fare for 2kms = Rs15+Rs8= Rs 23 = a_2

Taxi fare for 3kms= Rs23+Rs8= Rs 31 = a_3

Taxi fare for 4kms= Rs31+Rs8= Rs 39 = a_4

and so on

a_2-a_1 = Rs 23- Rs 15 = Rs8

a_3-a_2 = Rs 31- Rs 23 = Rs8

a_4-a_2 = Rs 39- Rs 31 = Rs8

i.e a_k+1-a_k is the same evrytime

So this list of numbers from an arithmetic progression with the first term **a=Rs15** and the common difference *d=Rs. 8/-*

(ii) Amount of air present in the cylinder = X units(say)

$= a_1$

Amount of air present in the cylinder after one time removal of air by the vacuum

$$Pamp = x - \frac{x}{4} = \frac{3x}{4} \ \ unit = a_2$$

Amount of air present in the cylinder after two time removal of air by the vacuum

$$Pamp = \frac{3x}{4} - \frac{x}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \left(\frac{3x}{16}\right) = \frac{9x}{16} units$$

$$= \left(\frac{3}{4}\right)^2 x \ \ units = a_3$$

Amount of air present in the cylinder after three time removal of air by the vacuum pamp

$$\left(\frac{3}{4}\right)^2 x = \frac{1}{4} - \left(\frac{3}{4}\right)^2 x$$

$$= \left(1 - \frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 x$$

$$= \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)x$$

$$= \left(\frac{3}{4}\right)^3 x \ units = a_4$$

and so on

$$a_2 - a_1 = \left(\frac{3x}{4}\right) - x = -\frac{x}{4} units$$

$$a_3 - a_2 = \left(\frac{3}{4}\right)^2 x - \frac{3}{4}x = -\frac{3}{16}x \ \ units$$

As a_2-$a_1 \neq a_3$ - a_2, this list of numbers does not from an AP

Soln:(iii)Cost of digging the well after 1 metre of digging =

= 150/- = a_1

Cost of digging the well after 2 metre of digging =

= Rs 150 + Rs 50 = Rs.200 = a_2

Cost of digging the well after 3 metre of digging =

= Rs. 200 + Rs. 50 = Rs.250 = a_3

Cost of digging the well after 4 metre of digging =

= Rs. 250 + Rs. 50 = Rs.300 = a_4

And So on

$$\Rightarrow a_2 - a_1 = Rs.\ 200 - Rs.\ 150 = Rs.50$$

$$\Rightarrow a_3 - a_2 = Rs.250 - Rs.200 = Rs.50$$

$$\Rightarrow a_4 - a_3 = Rs.300 - Rs.250 = Rs.50$$

i.e., $a_{n+1} - a_n$ is the same every time hence it is an A.P with common difference d = Rs. 50 and first term a = Rs. 150

(iv)

amount of money after 1 year

$$= Rs.10000\left(1 + \frac{8}{100}\right) = a_1$$

amount of money after 2 year

$$= Rs.10000\left(1 + \frac{8}{100}\right)^2 = a_2$$

amount of money after 3 year

$$= Rs.10000\left(1 + \frac{8}{100}\right)^3 = a_3$$

amount of money after 4 year

$$= Rs.10000\left(1 + \frac{8}{100}\right)^4 = a_4$$

$$a_2 - a_1 = Rs.10000\left(1 + \frac{8}{100}\right)^2 - Rs.10000\left(1 + \frac{8}{100}\right)$$

$$= Rs.10000\left(1 + \frac{8}{100}\right)\left(1 + \frac{8}{100} - 1\right)$$

$$a_3 - a_2 = Rs.10000\left(1 + \frac{8}{100}\right)^3 - Rs.10000\left(1 + \frac{8}{100}\right)^2$$

$$= Rs.10000\left(1 + \frac{8}{100}\right)^2\left(1 + \frac{8}{100} - 1\right)$$

$$= Rs.10000\left(1 + \frac{8}{100}\right)^2\left(\frac{8}{100}\right)$$

Here a_2-$a_1 \neq a_3$ - a_2

So the given list of number deos not from an AP

7. The first term of an A. P is 2 and common difference is 5. Find the A. P.

Soln:- The first term of an A. P is 2, this means 'a' is 2.

The common difference is 5, this means 'd' is 5.

$$a_1 = a = 2$$

$$a_2 = a_1 + d = 2 + 5 = 7$$

$$a_3 = a_2 + d = 7 + 5 = 12$$

$$a_4 = a_3 + d = 12 + 5 = 17$$

$\therefore$ the A. P is 2, 7, 12, 17.....

8. The angles of the triangle are in A. P. The smallest angle is 30^0. Show that the triangle is a right angled triangle.

Soln:- Let ABC be a Δle in which A is the smallest angle = 30^0

It is given that the angles of the triangle are in A. P, therefore the angles A, B and C are 30^0, 30^0 + d, $\mathbf{30^0 + 2d}$ respectively.,

The sum of the 3 angles of a triangle is 180^0.

Hence, $300 + 300 + d + 300 + 2d = 180^0$

$$\Rightarrow 90^0 + 3d = 180^0$$

$$3d = 180^0 - 90^0$$

$$\therefore d = \frac{90}{3} = 30^0$$

(Contd)