

Mathematics

Quadratic Equations

Exercise- 4.2

6) A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article

Soln:- (.....Contd)

$$x = -15/2, 6$$

Since, x is the number of articles, it can not be negative, So the number of articles produced on that day = 6

$$\begin{aligned} \text{The cost of each article} &= 2x + 3 \\ &= (2 \times 6) + 3 \\ &= 15 \end{aligned}$$

Exercise- 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

$$(i) 2x^2 - 3x + 5 = 0 \quad (ii) 3x^2 - 4\sqrt{3}x + 4 = 0$$

$$(iii) 2x^2 - 6x + 3 = 0$$

Soln:-

$$i) 2x^2 - 3x + 5 = 0$$

$$\text{Here } a = 2, b = -3, c = 5$$

$$\begin{aligned} \text{Therefore, discriminant} &= b^2 - 4ac \\ &= (-3)^2 - 4(2)(5) \\ &= 9 - 40 \\ &= -31 \end{aligned}$$

So, the given quadratic equation has no real roots

$$(ii) 3x^2 - 4\sqrt{3}x + 4 = 0$$

$$\text{Here } a = 3,$$

$$b = -4\sqrt{3}x$$

$$c = 4$$

Therefore, discriminant = $b^2 - 4ac$

$$\begin{aligned} &= (-4\sqrt{3}x)^2 - 4(3)(4) \\ &= 48 - 48 \\ &= 0 \end{aligned}$$

Hence the given quadratic equation has two equal real root

$$= \frac{b}{2a}, -\frac{b}{2a}$$

$$= \frac{(-4\sqrt{3}x)}{2 \times 3} - \frac{(-4\sqrt{3}x)}{2 \times 3} \text{ ie } \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

$$(iii) 2x^2 - 6x + 3 = 0$$

$$\text{Here } a = 2,$$

$$b = -6$$

$$c = 3$$

Therefore, discriminant = $b^2 - 4ac$

$$\begin{aligned} &= (-6)^2 - 4(2)(3) \\ &= 36 - 24 \\ &= 12 \end{aligned}$$

So, the quadratic equation has two distinct real roots
Solving the quadratic equation $2x^2 - 6x + 3 = 0$ by the quadratic formula we get

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} &= \frac{-6 \pm \sqrt{12}}{2(2)} \\ &= \frac{-6 \pm 2\sqrt{3}}{2} \\ &= \frac{-3 \pm \sqrt{3}}{2} \end{aligned}$$

Therefore, the roots are $\frac{-3 \pm \sqrt{3}}{2}$ ie $\frac{-3 + \sqrt{3}}{2}$ and $\frac{-3 - \sqrt{3}}{2}$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

$$(i) 2x^2 + kx + 3 = 0 \quad (ii) kx(x - 2) + 6 = 0$$

Soln:-

$$(i) 2x^2 + kx + 3 = 0$$

$$\text{Here } a = 2,$$

$$b = k$$

$$c = 3$$

Therefore, discriminant = $b^2 - 4ac$

$$\begin{aligned} &= k^2 - 4(2)(3) \\ &= k^2 - 24 \end{aligned}$$

If the given quadratic equation has two real equal roots then,

$$\begin{aligned} &b^2 - 4ac \\ \Rightarrow k^2 - 24 &= 0 \\ \Rightarrow k &= 24 \\ \Rightarrow k &= \pm\sqrt{24} \\ \Rightarrow k &= \pm 2\sqrt{6} \end{aligned}$$

hence, The required value of k are

$$\pm 2\sqrt{6} \text{ ie } 2\sqrt{6} \text{ and } -2\sqrt{6}$$

$$(ii) kx(x - 2) + 6 = 0$$

$$\text{Here } a = k,$$

$$b = -2k$$

$$c = 6$$

Therefore, discriminant = $b^2 - 4ac$

$$\begin{aligned} &= -(2k)^2 - 4(k)(6) \\ &= 4k^2 - 24k \end{aligned}$$

If the given quadratic equation has two real equal roots then,

$$\begin{aligned} &b^2 - 4ac \\ \Rightarrow 4k^2 - 24k &= 0 \\ \Rightarrow 4k(k - 6) &= 0 \\ \Rightarrow k(k - 6) &= 0 \\ \Rightarrow k &= 0, 6 \end{aligned}$$

$K=0$ is inadmissible as then the given quadratic equation reduced to $b=0$, which is not true

$$\therefore k = 6$$

Hence the required value of k is 6

3) Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2 ? If so, find its length and breadth.

Soln:- Let the breadth of the rectangular mango grove be $x \text{ m}$

Then the length of the rectangular mango grove = $2x \text{ m}$

Therefore

Area of the rectangular mango grove

$$= \text{Length} \times \text{Breadth}$$

$$= 2x \times x \text{ m}$$

According to the question

$$2x^2 = 800$$

$$x^2 = 800/2 = 400$$

$$x = \pm\sqrt{400}$$

$$x = \pm 20$$

$$x = +20, x = -20,$$

is in admissible as x is a dimension (Contd.....)