Mathematics : Real Numbers

3. **Find the LCM and HCF of the following integers by applying the prime factorisation method.**

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

Ans: (1) 12, 15 and 21

12 = 2 x2 x3

15 = 3 x 5

21 = 3 x 7

HCF = 3

LCM = 3 x2 x2 x5 x7 = 420

Ans: (ii) 17, 23 and 29

17 = 17 x 1

23 = 23 x 1

29 = 29 x 1

HCF. = 1

LCM = 17 x23 x29

= 11339

Ans: (iii) 8, 9 and 25

8 = 2 x 2 x 2= 2^3

9 = 3 x 3 = 3^2

25 = 5 x5 = 5^2

HCF. = 1

LCM =2^3 x3^2 x5^2

= 8 x9 x25

=1800

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Ans:– LCM (a, b) x HCF (a, b) = a x b

9 x LCM = 306x 657

$$\text{LCM} = \frac{306 \times 657}{9}$$

= 22338

(306, 657) HCF = 9 but their LCM = 22338

(306, 657) HCF = 9 but their LCM = 22338

5. Check whether 6n can end with the digit 0 for any natural number n.

Ans:- A number ends with 0 if it is divisible by 10. For a number to be divisible by 10, it must be divisible by both 2 and 5.

Divisibility by 2: Since 6 is divisible by 2, any multiple of 6 (6n) will also be divisible by2

Divisibility by 5: However, 6 is not divisible by 5. Therefore, no multiple of 6 (6n) can be divisible by 5.

Conclusion:Because 6n cannot be divisible by both 2 and 5, it cannot be divisible by 10. As a result, 6n cannot end with the digit 0 for any natural number n.

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Ans:- 7 X 11 x13 +13 = 13 (7 x 11 +1)

= 13 x78

∴ composite numbers

7 x6 x5 x4 x3 x2 x1 +5

= 5(7×6×4×3×2×1) +1

= 5(1008+1) = 5×1009

∴ composite numbers

In summary, both expressions are composite because they can be factored into two smaller factors other than 1 and themselves.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Ans:- To find when Sonia and Ravi will meet again at the starting point, we need to find the least common multiple (LCM) of their round times.

Sonia takes 18 minutes per round.

Ravi takes 12 minutes per round.

LCM of 18 and 12:

Prime factorization of 18:- 2 x 3 x 3

Prime factorization of 12:- 2 x 2 x 3

LCM (18, 12) = 2 x 2 x 3 x 3 = 36

Therefore, Sonia and Ravi will meet again at the starting point after 36 minutes.

EXERCISE 1.2

1. Prove that √5 is irrational.

Ans:- Let us assume, that √5 is rational number.

i.e. √5 = x/y (where, x and y are co-primes)

y√5= x

Squaring both the sides, we get,

$(y\sqrt{5})^2 = x^2$

$\Rightarrow 5y^2 = x^2$………………………………. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,

$5y^2 = (5k)^2$

$\Rightarrow y^2 = 5k^2$

is divisible by 5 it means y is divisible by 5.

Clearly, x and y are not co-primes. Thus, our assumption about √5 is rational is incorrect.

Hence, √5 is an irrational number.

2. Prove that 3+2√5 is irrational.

Ans:- Let us assume 3 + 2√5 is rational.

Then we can find co-prime x and y (y ≠ 0) such that 3 + 2√5 = x/y

Rearranging, we get,

$$2\sqrt{5} = \frac{x}{y} - 3$$

$$\sqrt{5} = \frac{1}{2}\left(\frac{x}{y} - 3\right)$$

Since, x and y are integers, thus,

$$\frac{1}{2}\left(\frac{x}{y} - 3\right)$$

is a rational number.

Therefore, √5 is also a rational number. But this contradicts the fact that √5 is irrational.

So, we conclude that 3 + 2√5 is irrational.

3. Prove that the following are irrationals :

(i) 1/ √2 (ii) 7√5 **(iii) 6 + √2**

Ans:- **(i) 1/√2**

Let us assume 1/√2 is rational. ***(Contd…)***