Mathematics

REVISION and MODEL QUESTIONS

6) A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8cm and 6cm, respectively (see Fig. 10.14). Find the sides AB and AC.

Soln: **(…..Contd)**

By putting the respective values, we get,

$2s = 28+2x$

$s = 14+x$

Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

By solving this, we get,

$= \sqrt{(14+x)48x}$ …… (i)

Again, the area of △ABC = 2 × area of (△AOF + △COD + △DOB)

$= 2\times[(½\times OF\times AF)+(½\times CD\times OD)+(½\times DB\times OD)]$

$= 2\times½(4x+24+32) = 56+4x$ ……..(ii)

Now from (i) and (ii), we get,

$\sqrt{(14+x)48x} = 56+4x$

Now, square both sides,

$48x(14+x) = (56+4x)^2$

$48x = [4(14+x)]^2/(14+x)$

$48x = 16(14+x)$

$48x = 224+16x$

$32x = 224$

$x = 7$ cm

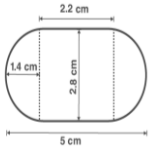
So, AB = 8+x

i.e. AB = 15 cm

And, CA = x+6 =13 cm.

7) A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and a diameter of 2.8 cm (see figure).

Soln:



It is known that the gulab jamuns are similar to a cylinder with two hemispherical ends.

So, the total height of a gulab jamun = 5 cm.

Diameter = 2.8 cm

So, radius = 1.4 cm

∴ The height of the cylindrical part =

= 5 cm–(1.4+1.4) cm

=2.2 cm

Now, the total volume of one gulab jamun =

=Volume of cylinder + Volume of two hemispheres

$= \pi r^2h+(4/3)\pi r^3$

$= 4.312\pi+(10.976/3)\,\pi$

$= 25.05$ cm^3

We know that the volume of sugar syrup = 30% of the total volume

So, the volume of sugar syrup in 45 gulab jamuns =

$= 45\times30\%(25.05\text{ cm}^3)$

$= 45\times7.515$

$= 338.184$ cm^3

8) **The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is 1/3. Find his present age.**

Soln: Let us say the present age of Rahman is x years.

Three years ago, Rehman's age was $(x – 3)$ years.

Five years after, his age will be $(x + 5)$ years.

Given the sum of the reciprocals of Rehman's ages 3 years ago and after 5 years is equal to 1/3.

$\therefore 1/x\text{-}3 + 1/x\text{-}5 = 1/3$

$(x+5+x\text{-}3)/(x\text{-}3)(x+5) = 1/3$

$(2x+2)/(x\text{-}3)(x+5) = 1/3$

$\Rightarrow 3(2x + 2) = (x\text{-}3)(x+5)$

$\Rightarrow 6x + 6 = x^2 + 2x – 15$

$\Rightarrow x^2 – 4x – 21 = 0$

$\Rightarrow x^2 – 7x + 3x – 21 = 0$

$\Rightarrow x(x – 7) + 3(x – 7) = 0$

$\Rightarrow (x – 7)(x + 3) = 0 \Rightarrow x = 7, -3$

As we know, age cannot be negative.

Therefore, Rahman's present age is 7 years

9) A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Sol^n:- Since each Prize is Rs 20 less than its preceding Prize, therefor, the values of the seven successive cash prizes will from an AP

Let the first prize be Rs a

Then the winner prizes in succession will be

Rs (a- 20), Rs(a-40), Rs(a-60) etc

Here a= ? d = (a-20) – a = 20

n= 7 Sn= 700

We know that

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$700 = \frac{7}{2}[2xa+(7-1)-20]$$

$$700 = \frac{7}{2}[2a-120]$$

$$700 = 7(a+60)$$

$$a-60 = \frac{700}{7}$$

$$a-60 = 100$$

$$a = 100+60$$

$$a = 160$$

Value of first price = Rs 160

Value of Second Prize

= Rs 160- Rs 20

=Rs 140

Value of third Prize

= Rs 140 - Rs 20 = Rs 120

Value of fourth Prize

= Rs 120 - Rs 20

=Rs 100

Value of fifth Prize

= Rs 100 - Rs 20

= Rs 80

Value of Sixth Prize

= Rs 80 - Rs 20

= Rs 60

Value of Seventh Prize

= Rs 60 - Rs 20 = Rs 40